A control approach to scheduling flexibly configurable jobs with dynamic structural-logical constraints

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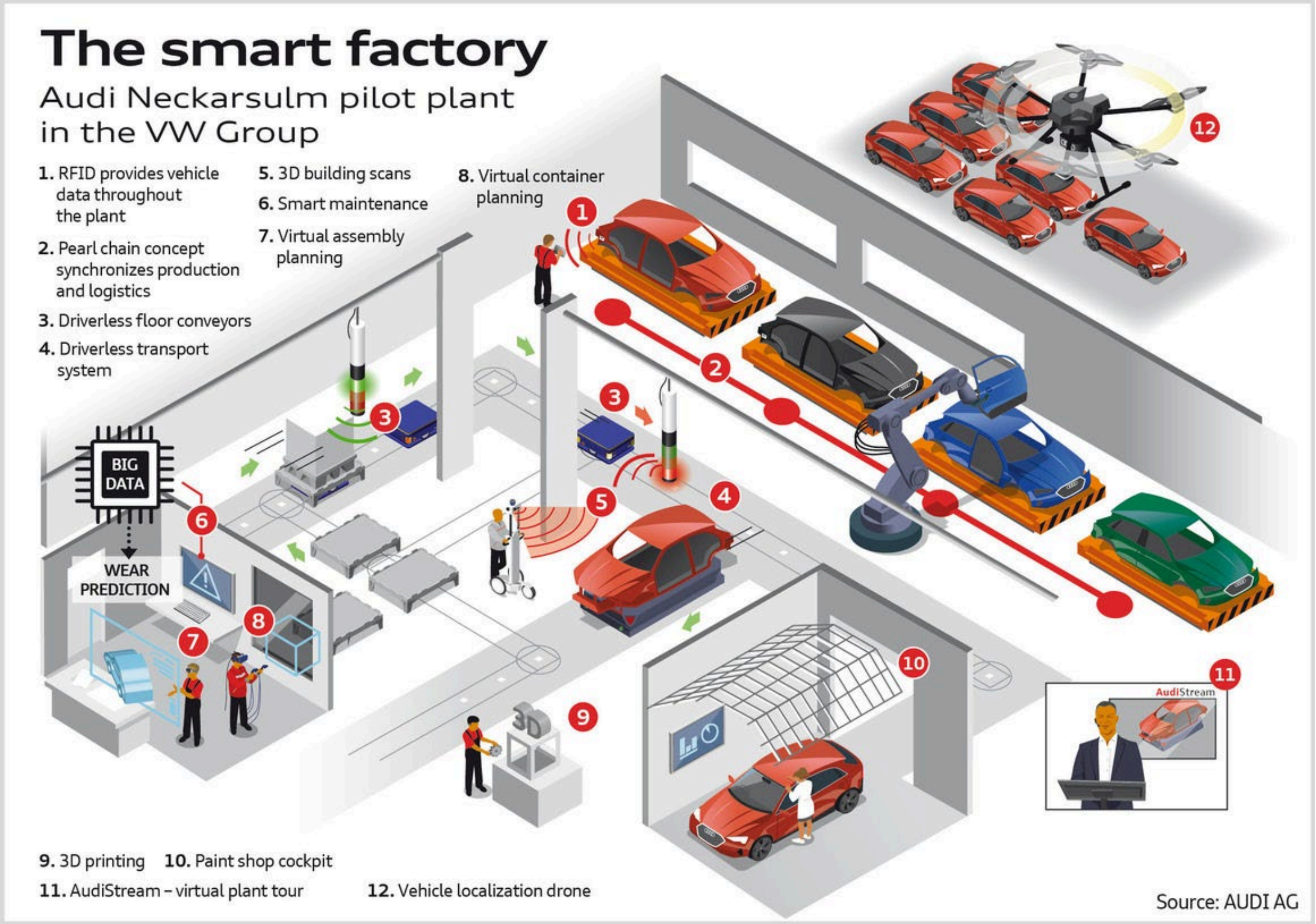
Outlines

- Problem description and industrial context
- Methodology: integrating optimal control and discrete optimization
- Model and algorithm
- Application
- Conclusions

Industrial context: Cloud manufacturing

New technologies enable highly flexible production, particularly through the use of cyber-physical systems and customized assemblies in order to deliver manufacturing services on-demand to consumers

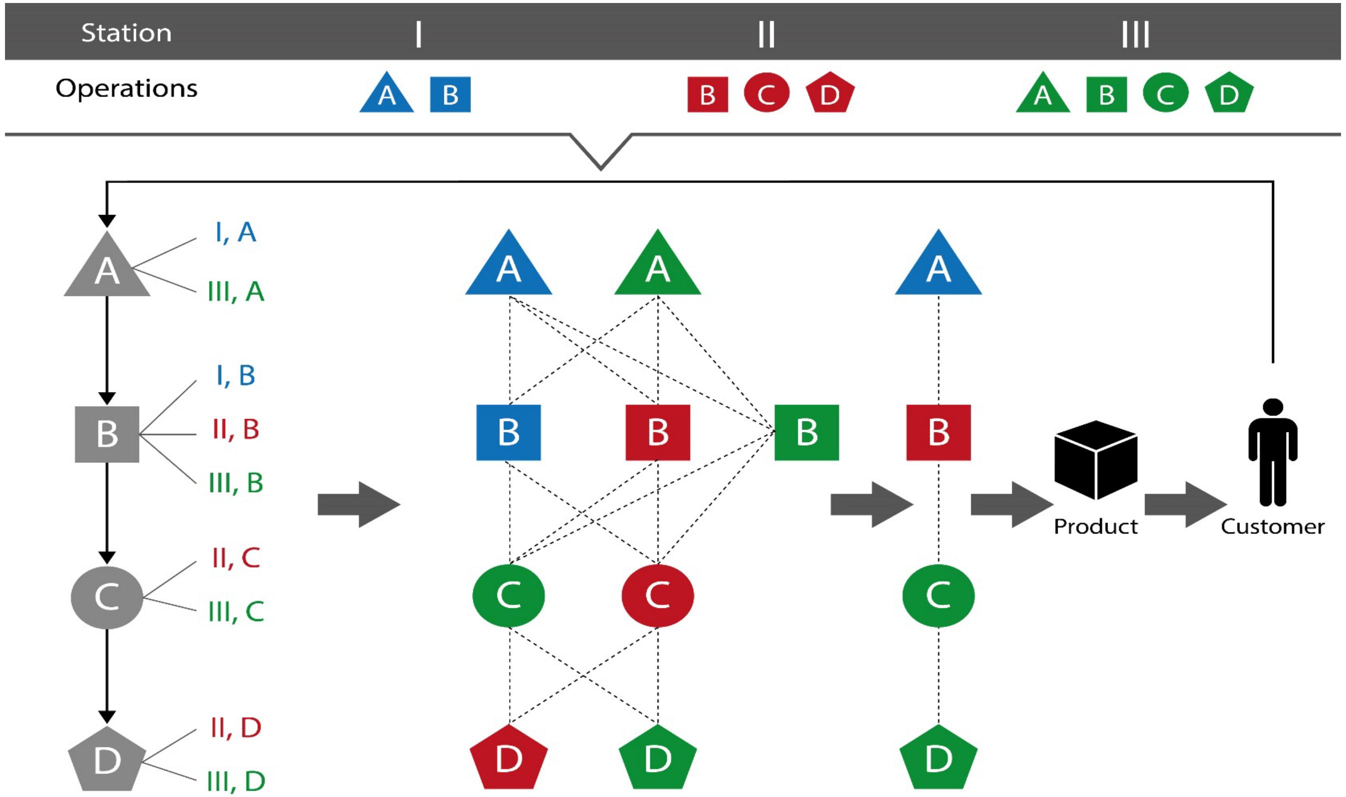
For example, the Audi R8 smart factory in Baden-Württemberg, Germany implements a highly flexible assembly system based on the use of automated guided vehicles. Contrary to the traditional assembly systems with fixed layouts and process designs, the Audi smart factory allows for highly flexible process design and sequencing of production orders in order to achieve the highest degree of the product individualization while maintaining the manufacturing efficiency.



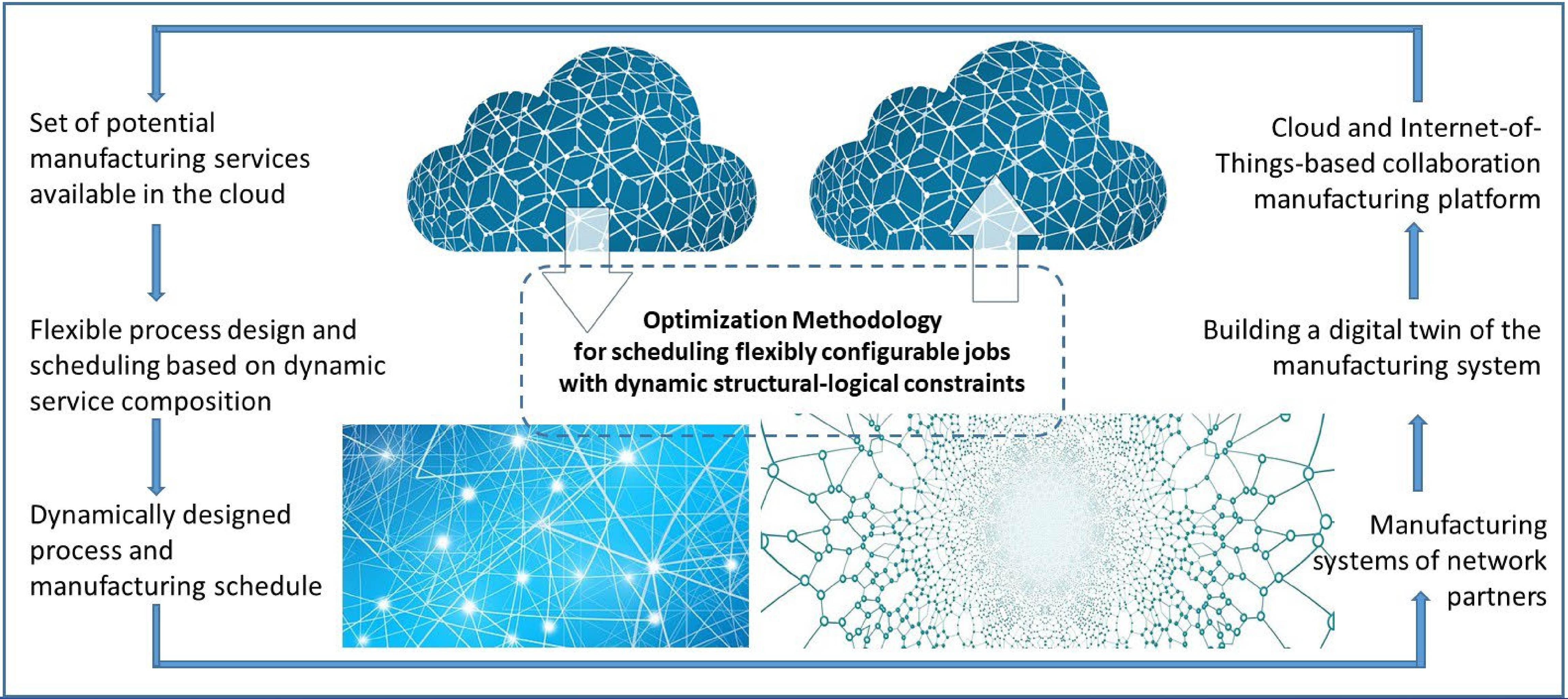
Industrial context: Cloud manufacturing

We study the problem of scheduling in manufacturing environments which are dynamically configurable for supporting highly flexible individual operation compositions of the jobs.

Such production environments yield the simultaneous process design and operation sequencing with dynamically changing hybrid structural-logical constraints.



Industrial context: Cloud Supply Chain



Challenge: simultaneous structural-functional synthesis of the process design and scheduling in systems with decentralized computational activities and dynamically changing resource availabilities

Methodology: integrating optimal control and discrete optimization

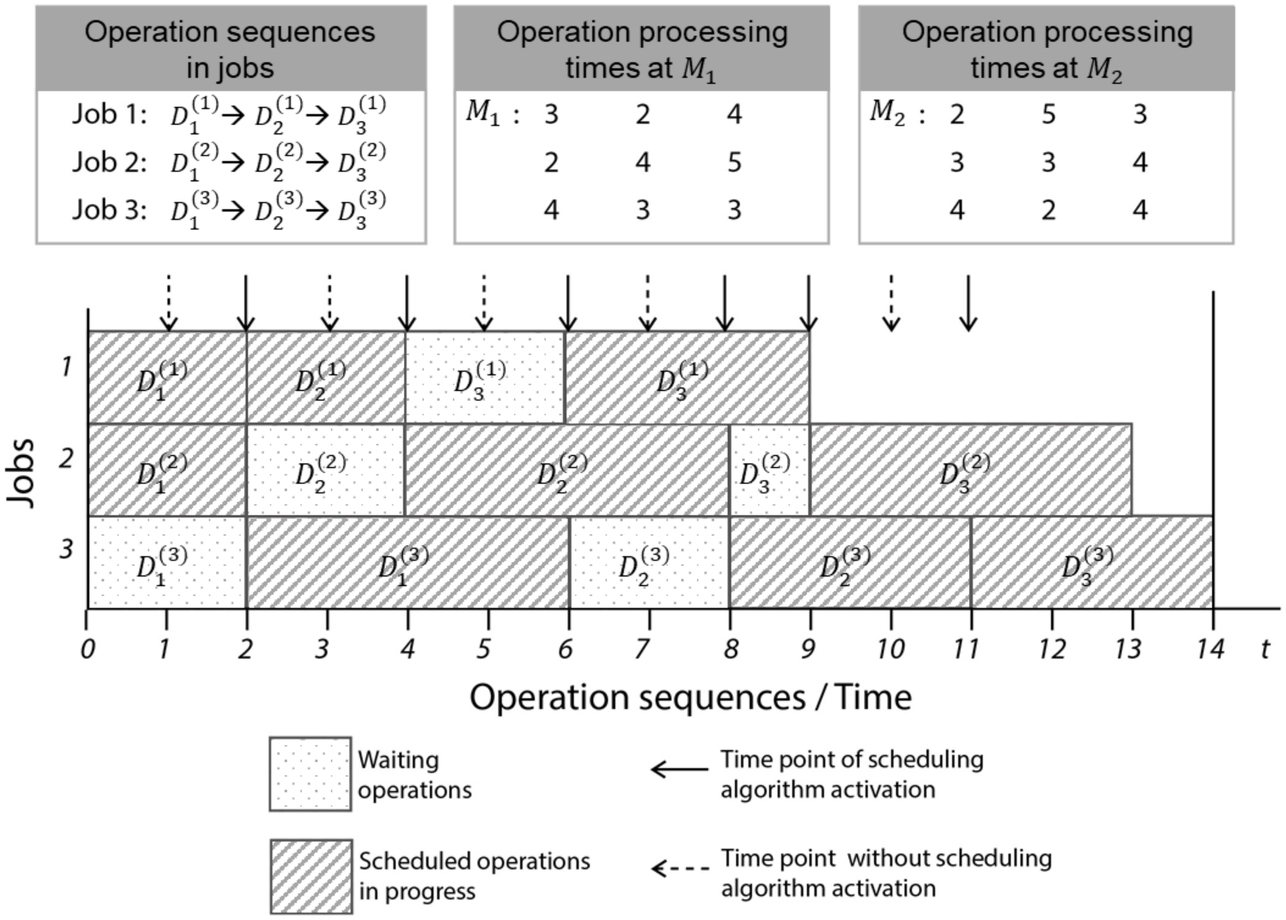
The methodology conceptualizes a control-based modeling approach to schedule flexible jobs in manufacturing systems when the structural-logical constraints are changing dynamically

We develop an optimal control model and algorithm for the simultaneous structural-functional design of a customized manufacturing process and the sequencing of the operations within the jobs, in the form of dynamically changing structural-logical constraints.

Our approach is explicitly capable of capturing dynamic features in flexible manufacturing with a simultaneous process design (i.e., task composition) and operation sequencing (i.e., service composition). Such dynamic scenarios are challenging to model in discrete optimization, but are convenient to describe using the continuous control paradigm to address the complexity of the dynamically changing hybrid structural-logical constraints by decomposition principles. Particularly, discrete optimization algorithms are used for scheduling in these matrices of small dimension at each time point. Continuous optimization algorithms are used to create a schedule from the discrete optimization results generated at each time point by extremizing the Hamiltonian function at each time point subject to some criteria.

Methodology: algorithmic realization

A distinctive feature of our approach is that we propose to decompose dynamically the large-scale assignment matrix according to the precedence relations between the operations of the jobs and dynamically consider only the operations that satisfy these precedence relations at a given time point in small-dimensional, discrete optimization models. Our method combines the advantages of continuous and discrete optimization.

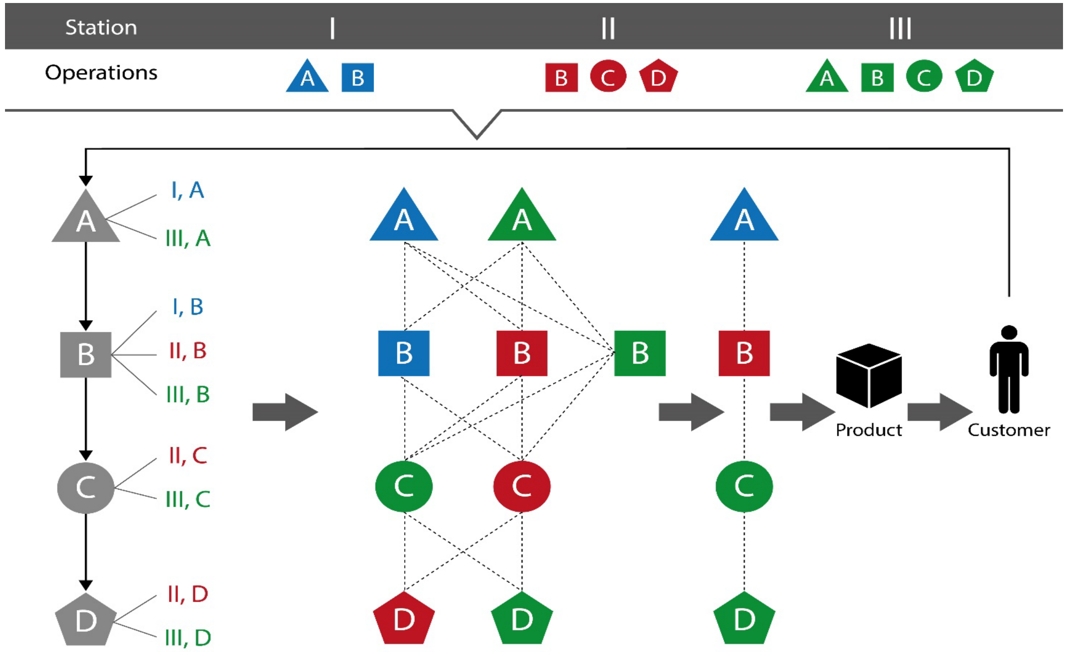


Problem description

The customer system generates orders (jobs) each of which has an individual sequence of the technological operations resulting in an individual *task composition*.

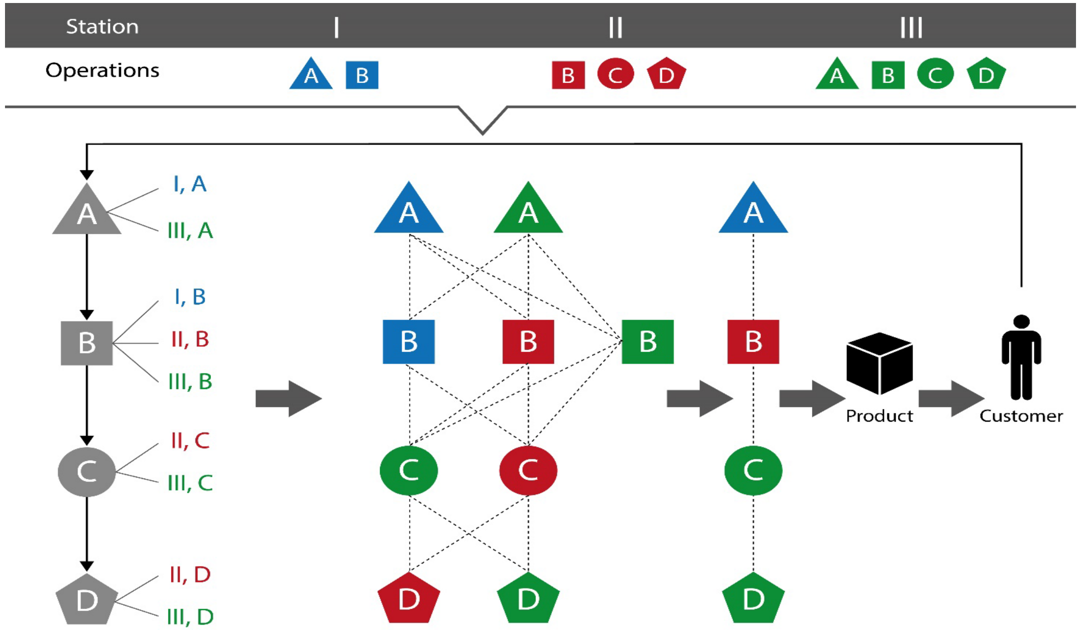
Each customer order can contain a unique chain of operations with the changing operation sequences in different orders

The first task is to design the manufacturing process, i.e., to perform a task composition by combining technological operations into a manufacturing process (i.e., the sequencing of operations into a process) Each of the operation sequences requires an individual *service composition*, i.e., a combination of an operation and a station. Thus, the second task is to implement a service composition by assigning the operations to stations at each stage of the technological process.



Problem description

The interactions of the customer and assembly systems result in alternatives for the design of the manufacturing process. Consequently, alternatives for job scheduling and sequencing exist, resulting in dynamic logical constraints which are, in turn subject to actual capacity utilization, machine availability, and time- and cost-related parameters of the services. As such, there are ***dynamic structural-logical constraints*** on the process design.

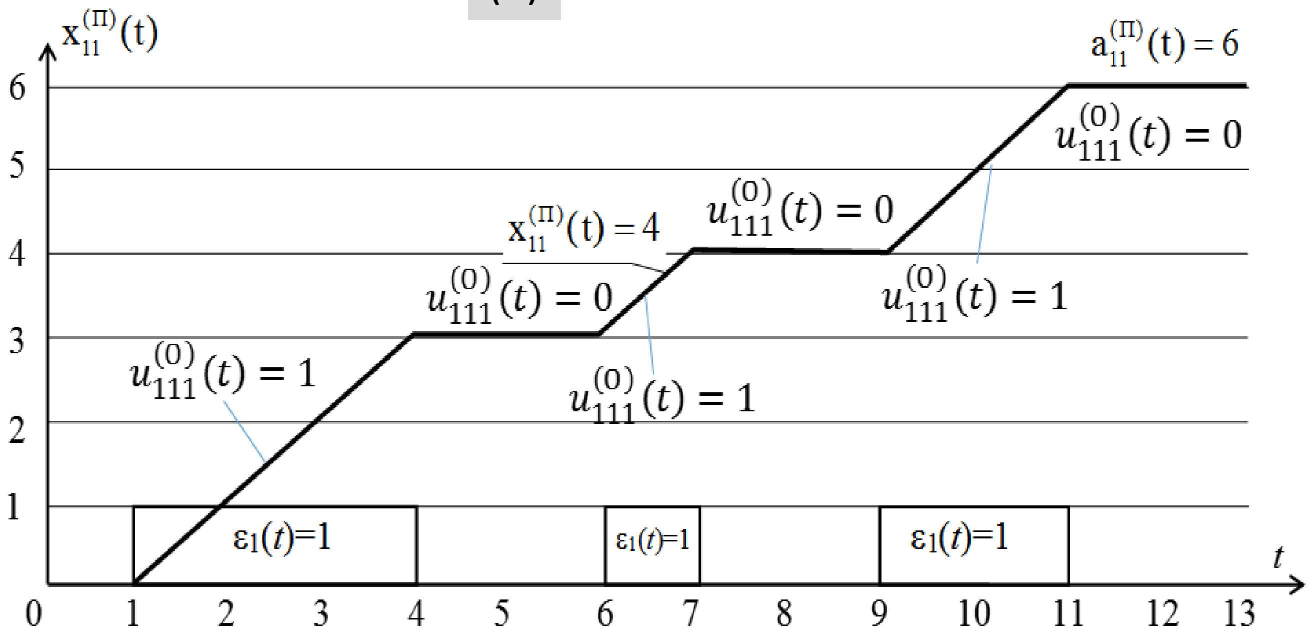


Process dynamics model

$$\dot{x}_{i\kappa}^{(O)} = \sum_{j=1}^{m} \varepsilon_j(t) \cdot \Theta_{i\kappa j}\left(u_{i\kappa j}^{(O)}(t) + \omega_{i\kappa j}^{(O)}(t)\right), \forall i, \kappa \quad (1)$$

$$\dot{x}_{i\kappa}^{(\Pi)} = \sum_{j=1}^{m} u_{i\kappa j}^{(\Pi)}(t), \forall i, \kappa \quad (2)$$

$$\dot{x}_{j}^{(P)} = \sum_{i=1}^{n} \sum_{\kappa=1}^{S_i} u_{i\kappa j}^{(O)}(t), \forall j \quad (3)$$



An example of a control profile for the execution of an operation ($k = 1$) at a machine ($m = 1$) is presented. The state variable $x_{11}^{(\Pi)}(t)$ accumulates the executed (processed) volume of the operation. Assuming that the planned execution volume is 6 units ($a_{i\kappa}^{(O)} = 6$), it can be observed from Fig. 3 that the operation can be completed (i.e.,$a_{i\kappa}^{(\Pi)} = x_{i\kappa}^{(\Pi)}$) at the given machine with a flow time of 11 time units.

The control variable $u_{111}^{(O)}(t)$ in Eq. (1) switches to 1 when the machine is available (i.e., $\varepsilon_j(t) = 1$) and the processed voume of the operation increases as reflected in $x_{11}^{(\Pi)}(t)$, e.g., $x_{11}^{(\Pi)}(t) = 4$ at $t = 7$. Eq. (3) describes the dynamics of the station utilization by means of the state variable $\dot{x}_j^{(P)}$. In case of $u_{i\kappa j}^{(O)}(t) = 0$, the station does not produce anything at time t. Similarly, if $u_{i\kappa j}^{(\Pi)}(t) = 0$, the product is not processed at time t.

Process dynamics model

$$\dot{x}_{i\kappa}^{(\mathrm{O})} = \sum_{j=1}^{m} \varepsilon_j(t) \cdot \Theta_{i\kappa j}\left(u_{i\kappa j}^{(\mathrm{O})}(t) + \omega_{i\kappa j}^{(\mathrm{O})}(t)\right), \forall i, \kappa \quad (1)$$

$$\dot{x}_{i\kappa}^{(\Pi)} = \sum_{j=1}^{m} u_{i\kappa j}^{(\Pi)}(t), \forall i, \kappa \quad (2)$$

$$\dot{x}_{j}^{(\mathrm{P})} = \sum_{i=1}^{n} \sum_{\kappa=1}^{S_i} u_{i\kappa j}^{(\mathrm{O})}(t), \forall j \quad (3)$$

Eqs. (1)-(3) are interconnected. While the process design (i.e., the technology synthesis) is described by Eq. (1), the sequencing is controlled by Eq. (2) by adjusting the flow time through the selection of processing intensities which allows to extremize the objective function. At the same time, the results of the process design directly affect the machine utilization using Eq. (3). In other words, Eqs. (1)-(3) describe the dynamic structural-functional synthesis of a flexible manufacturing system.

The state variables $\dot{x}_{i\kappa}^{(o)}$ and $\dot{x}_{i\kappa}^{(\Pi)}$ accumulate the processed quantities/volumes (e.g., a production output) at each point of time. This is one of the advantageous of continuous optimization-based modelling since optimal control is a convenient way both to develop supply chain process optimization models in terms of maximizing some output for some dynamically changing input and to describe the dynamics of process fulfillment at each point of time.

Structural-logical constraints

$$\sum_{j=1}^{m} u^{(O)}_{i\bar{\alpha}j}\cdot\sum_{\kappa\in\Gamma^{+}_{i\bar{\alpha}}} x^{(O)}_{i\kappa} = 0, \forall i, \bar{\alpha} \qquad (4)$$

$$\sum_{j=1}^{m} u^{(O)}_{i\bar{\beta}j}\cdot\prod_{\kappa\in\Gamma^{+}_{i\bar{\beta}}} x^{(O)}_{i\kappa} = 0, \forall i, \bar{\beta} \qquad (5)$$

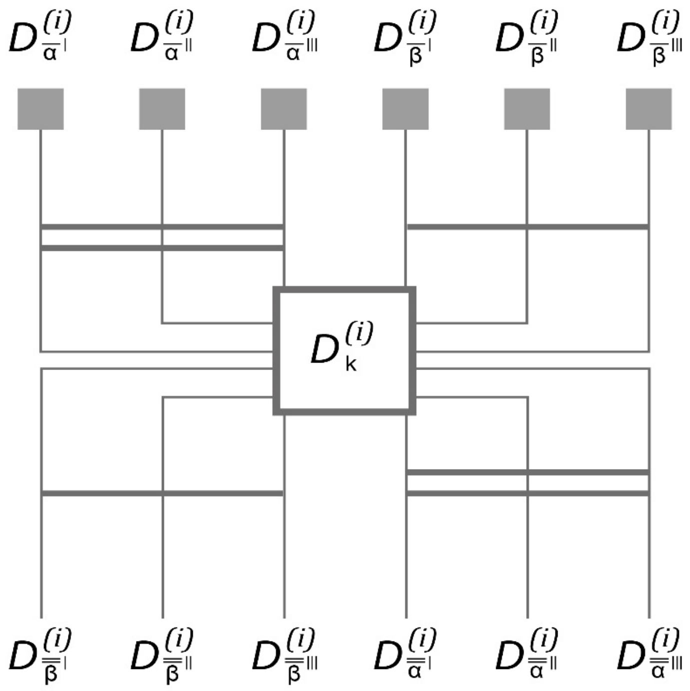
$$\sum_{j=1}^{m} u^{(O)}_{i\bar{\bar{\alpha}}j}\cdot\sum_{\kappa\in\Gamma^{-}_{i\bar{\bar{\alpha}}}} \left(a^{(\Pi)}_{i\kappa} - x^{(\Pi)}_{i\kappa}\right) = 0, \forall i, \bar{\bar{\alpha}} \qquad (6)$$

$$\sum_{j=1}^{m} u^{(O)}_{i\bar{\bar{\beta}}j}\cdot\prod_{\kappa\in\Gamma^{-}_{i\bar{\bar{\beta}}}} \left(a^{(\Pi)}_{i\kappa} - x^{(\Pi)}_{i\kappa}\right) = 0, \forall i, \bar{\bar{\beta}} \qquad (7)$$

$$\sum_{j=1}^{m} \omega^{(O)}_{i\kappa j}\left[a^{(\Pi)}_{i\kappa} - x^{(\Pi)}_{i\kappa}\right] = 0, \forall i, \kappa \qquad (8)$$

$$0 \le u^{(\Pi)}_{i\kappa j} \le c^{(\Pi)}_{i\kappa j} u^{(O)}_{i\kappa j}(t)\xi_j(t), \forall i, \kappa, j \qquad (9)$$

The structural-logical constraints are represented by Eqs. (4)-(9), which change dynamically.



Eqs. (4) and (5) - precedence relations for operation $D^{(i)}_{\kappa}$ with regard to the predecessor operations $D^{(i)}_{\bar{\alpha}}$ and $D^{(i)}_{\bar{\beta}}$. (6) and (7) - precedence relations for operation $D^{(i)}_{\kappa}$ with regard to the subsequent operations $D^{(i)}_{\bar{\bar{\alpha}}}$ and $D^{(i)}_{\bar{\bar{\beta}}}$. (8) - the logic for the auxiliary control variable $\omega^{(O)}_{i\kappa j} \in \{0,1\}$ which equals 1 if $x^{(\Pi)}_{i\kappa}(t) = a^{(\Pi)}_{i\kappa}$ at time point t and equals 0 if $x^{(O)}_{i\kappa} \neq a^{(O)}_{i\kappa}$. In other words, the flow processing is completed. (9) - processing capacity constraint.

Structural-logical constraints

$$\sum_{j=1}^{m} u_{i\bar{\alpha}j}^{(\text{O})} \cdot \sum_{\kappa \in \Gamma_{i\bar{\alpha}}^{+}} x_{i\kappa}^{(\text{O})} = 0, \forall i, \bar{\alpha} \qquad (4)$$

$$\sum_{j=1}^{m} u_{i\bar{\beta}j}^{(\text{O})} \cdot \prod_{\kappa \in \Gamma_{i\bar{\beta}}^{+}} x_{i\kappa}^{(\text{O})} = 0, \forall i, \bar{\beta} \qquad (5)$$

$$\sum_{j=1}^{m} u_{i\bar{\bar{\alpha}}j}^{(\text{O})} \cdot \sum_{\kappa \in \Gamma_{i\bar{\bar{\alpha}}}^{-}} \left(a_{i\kappa}^{(\Pi)} - x_{i\kappa}^{(\Pi)}\right) = 0, \forall i, \bar{\bar{\alpha}} \qquad (6)$$

$$\sum_{j=1}^{m} u_{i\bar{\bar{\beta}}j}^{(\text{O})} \cdot \prod_{\kappa \in \Gamma_{i\bar{\bar{\beta}}}^{-}} \left(a_{i\kappa}^{(\Pi)} - x_{i\kappa}^{(\Pi)}\right) = 0, \forall i, \bar{\bar{\beta}} \qquad (7)$$

$$\sum_{j=1}^{m} \omega_{i\kappa j}^{(\text{O})} \left[a_{i\kappa}^{(\Pi)} - x_{i\kappa}^{(\Pi)}\right] = 0, \forall i, \kappa \qquad (8)$$

Eqs. (4)-(9) can be considered as active dynamic constraints meaning that the number of operations in those constraints is changing dynamically in time in relation to Eqs. (1)-(3). Along with the operations, flow and station dynamics in the process control model (1)-(3), the dynamic changes in constraints (4)-(9) determine the dimensionality of the scheduling problem in a discrete optimization model at each t.

Remark 1. The expressions in Eqs. (4)-(8) are equal to zero if, and only if the control variables are equal to 1, which means that all the predecessor operations have been executed. This leads by tendency to a constraint system of small dimensionality at each point of time that can be solved using discrete optimization techniques of integer and linear programming.

Algorithm

$$H(\boldsymbol{x}(t), \boldsymbol{u}(t), \boldsymbol{\psi}(t)) = H_1 + H_2 + H_3 + H_4 + H_5 + H_6 + H_7,$$

where

$$H_1 = \sum_i \sum_j \sum_\mu \left[\psi_{i\kappa}^{(O)} \varepsilon_{i\kappa j}(t) \Theta_{i\kappa j} + \psi_j^{(P)} + \varphi_{i\kappa j}^{(1)}\right] \cdot u_{i\kappa j}^{(O)}, \tag{28}$$

$$H_2 = \sum_i \sum_\mu \sum_j \left[\psi_{i\kappa}^{(\Pi)}\right] \cdot u_{i\kappa j}^{(\Pi)}, \tag{29}$$

$$H_3 = \sum_i \sum_\kappa \sum_j \left[\varphi_{i\kappa j}^{(1)} + \psi_{i\kappa}^{(O)} \cdot \varepsilon_{i\kappa j} \cdot \Theta_{i\kappa j}\right] \cdot \omega_{i\kappa j}^{(O)}, \tag{30}$$

$$H_4 = \sum_i \sum_\kappa \sum_j \varphi_{i\kappa j}^{(2)} \cdot z_{i\kappa j}; \;\; H_5 = \sum_i \sum_\kappa \sum_j \varphi_{i\kappa j}^{(3)} \cdot \nu_{i\kappa j}, \tag{31}$$

$$H_6 = \sum_i \sum_\kappa p_{i\kappa}^{(B,1)} \cdot u_{i\kappa}^{(B,1)}; \;\; H_7 = \sum_i \sum_\mu p_{i\kappa}^{(B,2)} \cdot u_{i\kappa}^{(B,2)}, \tag{32}$$

where $\boldsymbol{\psi}(t)$ is the adjoint vector.

Algorithm 1: DYN-CONTROL

0. Set $t = 0$, and initialize all parameters.
1. From the time point $t = t_0$ onwards, determine the control $\boldsymbol{u}^{(r+1)}(t)$, where $r = 0, 1, 2, \ldots$ denotes the number of the iteration.
2. For the given initial boundary conditions (21) and $\boldsymbol{\psi}(t_0)$, compute control vector $\boldsymbol{u}^*(t)$ at $t = t_0$ to maximize (28)-(32).

   (a) Solve the assignment problem: solve the maximization of the Hamiltonian H_1 for the model (1), (3) with the constraints (7), (20).
   (b) Solve the linear programming problem: solve the maximization of the Hamiltonian H_2 for the model (2) with the constraints (15), (19).
   (c) Operation selection: select the operations and constraints which meet the requirements (7), (19), (20).

Algorithm

$$H(\boldsymbol{x}(t),\boldsymbol{u}(t),\boldsymbol{\psi}(t)) = H_1 + H_2 + H_3 + H_4 + H_5 + H_6 + H_7,$$

where

$$H_1 = \sum_i \sum_j \sum_\mu \left[\psi_{i\kappa}^{(O)} \varepsilon_{i\kappa j}(t) \Theta_{i\kappa j} + \psi_j^{(P)} + \varphi_{i\kappa j}^{(1)}\right] \cdot u_{i\kappa j}^{(O)}, \tag{28}$$

$$H_2 = \sum_i \sum_\mu \sum_j \left[\psi_{i\kappa}^{(\Pi)}\right] \cdot u_{i\kappa j}^{(\Pi)}, \tag{29}$$

$$H_3 = \sum_i \sum_\kappa \sum_j \left[\varphi_{i\kappa j}^{(1)} + \psi_{i\kappa}^{(O)} \cdot \varepsilon_{i\kappa j} \cdot \Theta_{i\kappa j}\right] \cdot \omega_{i\kappa j}^{(O)}, \tag{30}$$

$$H_4 = \sum_i \sum_\kappa \sum_j \varphi_{i\kappa j}^{(2)} \cdot z_{i\kappa j};\ \ H_5 = \sum_i \sum_\kappa \sum_j \varphi_{i\kappa j}^{(3)} \cdot \nu_{i\kappa j}, \tag{31}$$

$$H_6 = \sum_i \sum_\kappa p_{i\kappa}^{(B,1)} \cdot u_{i\kappa}^{(B,1)};\ \ H_7 = \sum_i \sum_\mu p_{i\kappa}^{(B,2)} \cdot u_{i\kappa}^{(B,2)}, \tag{32}$$

where $\boldsymbol{\psi}(t)$ is the adjoint vector.

3. $\boldsymbol{u}^*(t_0)$ is then put into (1)-(6) and (33)–(39).
4. The main and adjoint systems are integrated using a forward integration procedure.
5. The transversality conditions (51)–(58) are evaluated.
6. Update $t \to t_1$.
7. $\boldsymbol{u}^*(t_1)$ is computed based on the Hamiltonian (28)-(32) maximization, where $t_1 = t_0 + \Delta$ (Δ is the chosen integration step). $\boldsymbol{x}^{(r)}(t_1)$ is generated.
8. If $t = t_f$, then the objective functions (23)-(27) are evaluated and the record value $J_G = J_G^{(r)}$ can be calculated, where J_G is a scalar form of the multi-criteria vector (23)-(27).
9. The integration process is continued until the end boundary conditions (22) are reached and the convergence for J_G is realized (i.e., no further improvement for J_G).
10. The resulted $\boldsymbol{u}^*(t)$ is the optimized schedule.

Industrial application: realization

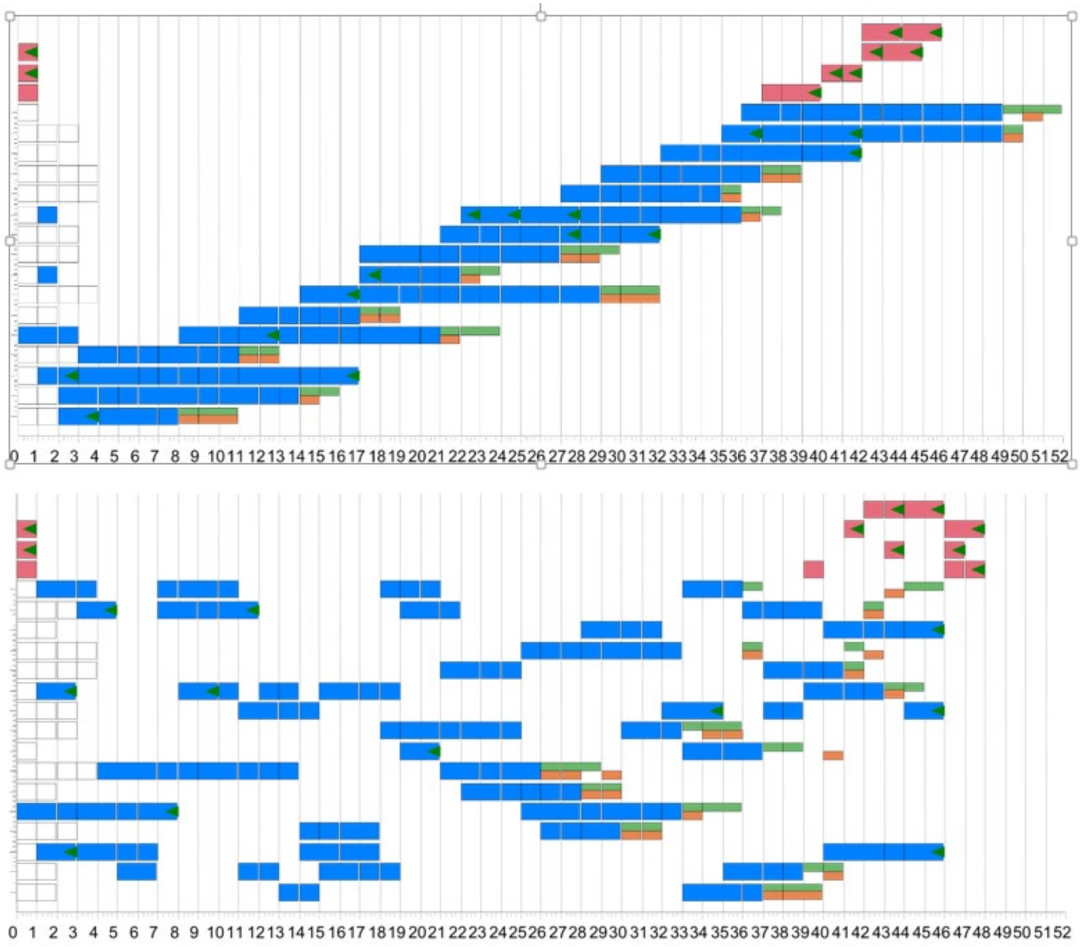
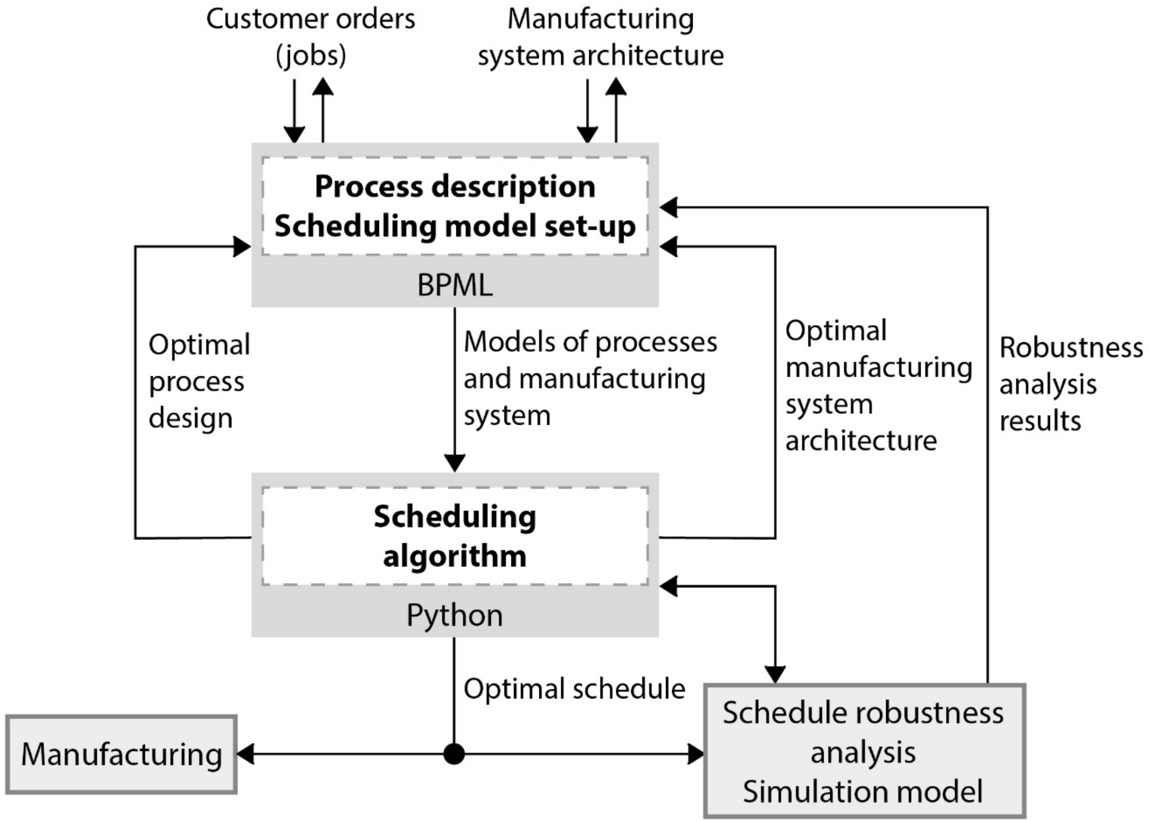
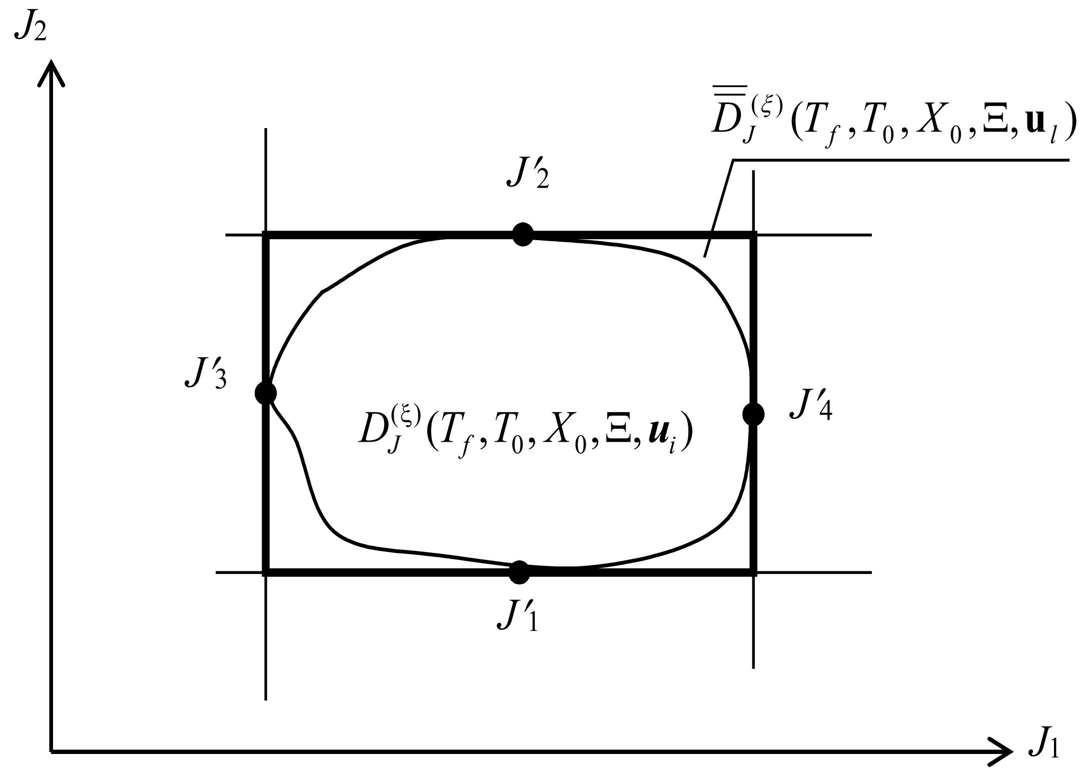
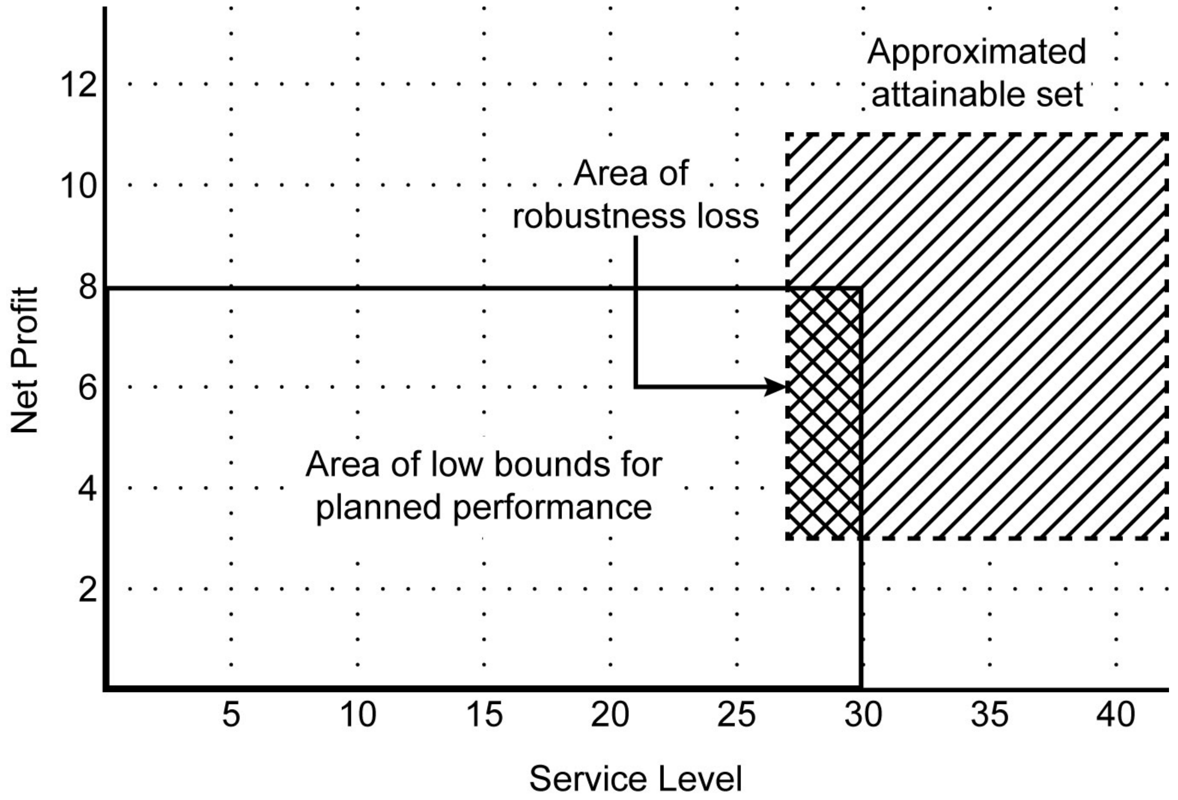


Fig. 10. Initial (top) and optimized (bottom) schedules



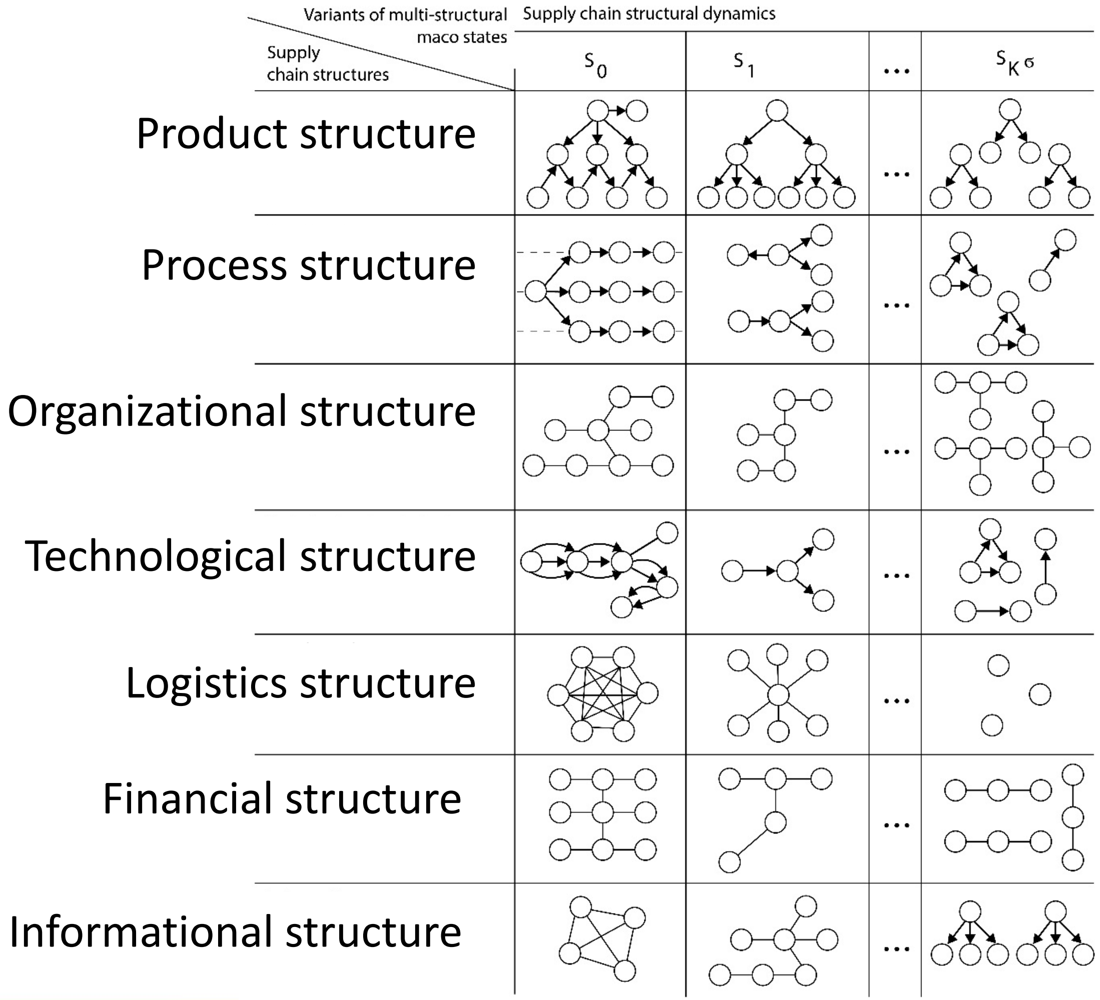
Some extensions: Robustness analysis with attainable sets





a)

Some extensions: Reconfigurable supply chain



The reconfigurable supply chain adds three specific features to RMS:

- active, goal-oriented *behavior* of network elements,
- *networking* effects across multiple structures and their dynamics (i.e., organizational, information, financial, technological, and energy), and
- network *complexity* (i.e., multi-echelon supply chains).

Ivanov, D. (2018). *Structural Dynamics and Resilience in Supply Chain Risk Management*. Springer, New York

Dolgui, A., Ivanov, D., Sokolov, B. (2020) Reconfigurable supply chain: The X-Network. **International Journal of Production Research**, 58(13), 4138-4163.

Summary

- Industry 4.0, smart manufacturing and reconfigurable manufacturing systems impose problems with simultaneous process design and scheduling leading to **dynamically changing structural-logical constraints:** product and process are created simultaneously
- Our methodological contribution is an approach to solving such problems using a **combination of optimal control and discrete optimization** – we build on synergy effects which allow to use advantages of one method to compensate for disadvantages of another one
- The combined approach allows both determining **discrete** start and end times for job processing and model dynamics of job processing in **continuous** time
- Using an original interpretation of job processing dynamics representation by optimal control, our approach is based on a **dynamic decomposition** of the assignment matrix using natural logic of time
- At each point of time, small-dimensional discrete optimization problems of (by tendency) polynomial complexity are formed and solved, and these partial solutions are **integrated** through an original algorithmic procedure based on Maximum principle
- **Sensitivity analysis** and **industrial application** validate the proposed approach.
- **Future areas:** *uncertainty* modelling; embedding into the real Industry 4.0 systems to feed *real-time data* into the constraint system parameters and to extract modelling results (e.g., values of state variables)

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Q&A

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