Micro-Macro Modelling of Piezoelectric Fiber Composites

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The present work deals with the numerical modeling of 1-3 periodic composites made of piezoceramic (PZT) fibers embedded in a soft non-piezoelectric matrix. We especially focus on predicting the effective co-efficients of the periodic transversely isotropic piezoelectric fiber composites using representative volume element method (unit cell method). The results which are obtained from the FEM technique are compared with analytical homogenization method for different volume fractions. The effective co-efficients are obtained for rectangular and hexagonal arrangement of unidirectional piezoelectric fiber composites.

1 Introduction

Piezoelectric materials have the property of converting electrical energy into mechanical energy, and vice versa. This reciprocity in the energy conversion makes piezoelectric ceramics such as PZT (Lead, Zirconium, Titanat) very attractive materials towards sensors and actuators applications. Even if their properties make them interesting, they are often limited, first by their weight, that can be a clear disadvantage for shape control and as a consequence, by their high specific acoustic impedance, which reduces their acoustic matching with the external fluid domain. Bulk piezoelectric materials have several drawbacks, hence composite materials are often a better technological solution in the case of a lot of applications such as ultrasonic transducers, medical imaging, sensors and actuators. A number of methods have been developed to predict and simulate the linear coupled piezoelectric and mechanical behavior of composites. Numerical and analytical approaches were made to estimate the effective coefficients. Several approaches have been developed to estimate the effective coefficients with numerical methods especially by means of finite element method (FEM), Gaudenzi (1997). With a so-called representative volume element (RVE) or unit cell method the problem can be reduced on investigation of a part of an infinite structure. In this case special boundary conditions must be applied to the RVE to ensure the periodicity in the deformation, Poizat et al. (1999), Petterman et al. (2000). Difficulties arise from right combination of displacement boundary conditions with special load cases to calculate the different effective coefficients as well as an appropriate finite element modeling.

2 Numerical Homogenization Techniques

The present work deals with the numerical modeling of 1-3 periodic composites made of piezoceramic (PZT-5A) fibers embedded in a soft non-piezoelectric matrix (polymer with isotropic elastic properties). The goal is to predict the full set of effective coefficients for periodic transversely isotropic piezoelectric fiber composites. This can be achieved by analyzing a RVE of the infinite composite (see figure 1) by means of FEM (FE code ANSYS was used). Using appropriate periodic boundary conditions and subjecting the RVE to different load cases provide the possibility for calculation of the effective coefficients. The criterion is equality in strain energy for composite body and for homogenized body. The results which are obtained from the FEM technique are compared with the analytical homogenization method for different volume fractions.

![Rectangular arrangement](image1.png)
![Hexagonal arrangement](image2.png)

Fig. 1 Piezo fiber composite and representative volume element (RVE).

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3 Piezoelectric Material Behavior

This paper considers piezoelectric materials that respond linearly to changes in the electric field, electric displacement, or mechanical stress and strain. These assumptions are compatible with the piezoelectric ceramics, polymers, and composites in current use. Therefore, the behavior of the piezoelectric medium is described by the following piezoelectric constitutive equations which relate the stress $T_{ij}$, strain $S_{kl}$, electric field $E_k$, and electrical displacement $D_i$ as $T_{ij} = C_{ijkl} S_{kl} - e_{kij} E_k$ and $D_i = e_{ikl} S_{kl} + \varepsilon_{ij} E_k$ where $C_{ijkl}$ is a fourth-order elasticity tensor under short circuit boundary conditions, $e_{kij}$ is a second-order free body dielectric tensor, and $e_{kij}$ is a third-order piezoelectric strain tensor.

Due to symmetry of the tensors $T_{ij}$, $S_{ij}$, $C_{ijkl}$ and $\varepsilon_{ij}$, the above equations can be written in a vector/matrix form as

$$
\begin{bmatrix}
T \\
D
\end{bmatrix} = 
\begin{bmatrix}
C & -e^T \\
\varepsilon & \varepsilon
\end{bmatrix} 
\begin{bmatrix}
S \\
E
\end{bmatrix}
$$
or
$$
\Psi = J \cdot \gamma,
$$

where superscript $T$ denotes a transposed matrix.

For a transversely isotropic piezoelectric solid, the stiffness matrix, the piezoelectric matrix and the dielectric matrix simplify so that there remain 11 independent coefficients. These coefficients must be calculated for the homogenized material which has the same overall behavior like the composite material. Insuring the periodicity implies that each RVE in the composite has the same deformation mode and there is no separation or overlap between the neighboring RVEs. These periodic displacement boundary conditions $u_i$ on opposite surfaces are given by $u_i = S_{ij} x_j + v_i$.

In the above equations $S_{ij}$ are the average strains, $v_i$ is the periodic part of the displacement components (local fluctuation) on the boundary surfaces which is generally unknown and is dependent on the applied global loads.

4 Results and Conclusions

The calculations of effective coefficients were made for different volume fractions and were compared with values obtained from analytical homogenization method reported e.g. in Berger et al. (2003). Figure 2 show the comparison for two selected effective coefficients. The numerical calculated effective coefficients are in good agreement with the analytical solution. Also the other coefficients not reported here show a similar tendency. The introduced numerical method for calculating effective coefficients of piezoelectric composites by means of FEM provide the basis to extend it to composites with a more complex distribution of embedded piezoelectric fibers or other geometric forms of piezoelectric inclusions.

References